

$$r_{\text{critical}} = \frac{1}{C_5} \left[ \left( \frac{7k}{8} - k_f \right) \pm \left[ \left( \frac{7k}{8} - k_f \right)^2 + k_f (k - k_f) \right]^{1/2} \right]^{4/3} \quad (14)$$

where

$$C_5 = 0.196 k_f \left[ \frac{8g\beta(T_o - T_\infty)}{\nu^2} \right]^{1/4} \quad (15)$$

Equations (10) and (14) reduce to the form of Equations (3) and (4), only when the additive constants in Equations (5) and (6) (0.6 and 2.0, respectively) are negligible when compared to the term containing the Grashof number. Equations (10) and (14) are, therefore, somewhat more general than Equations (3) and (4).

Simmons (1976) used Equation (2) in determining the critical radius for combined convection and radiation. However, the algebraic simplification that resulted from the use of Equation (2) in his analysis does not occur with the use of Equations (7) and (8). Consequently, the use of these more general convective heat transfer coefficient correlations in Simmons' analysis does not produce an explicit solution for the critical radius in the case of combined convection and radiation.

#### NOTATION

$C_1, C_2, C_3, C_4$  = constants in Equations (7) and (8)  
 $Gr$  = Grashof number  
 $h$  = convective heat transfer coefficient ( $\text{Wm}^{-2} \text{ } ^\circ\text{C}^{-1}$ )  
 $H$  = constant in Equation (2)

$k$  = thermal conductivity of object ( $\text{Wm}^{-1} \text{ } ^\circ\text{C}^{-1}$ )  
 $k_f$  = thermal conductivity of surrounding fluid ( $\text{Wm}^{-1} \text{ } ^\circ\text{C}^{-1}$ )  
 $m, n$  = constants in Equation (2) ( $\geq 0$ )  
 $Pr$  = Prandtl number  
 $q$  = heat transfer (W)  
 $r$  = radius of the object (m)  
 $T_o$  = surface temperature of the object ( $^\circ\text{C}$ )  
 $T_\infty$  = fluid temperature ( $^\circ\text{C}$ )

#### LITERATURE CITED

- Balmer, R. T., and A. D. Strobusch, "Critical Size of Newborn Homeotherms," *J. Appl. Phys.* **42**, 571 (1977). [Note that Equations (31), (37), (38), (39), (40), and (42) are incorrect as a result of an error in Equation (31). This error has been corrected in the present note.]  
Churchill, S. W., and H. H. Chu, "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder," *Intern. J. Heat Mass Transfer*, **18**, 1049 (1975).  
Kreith, F., *Principles of Heat Transfer*, pp. 37-58, International Textbook, Scranton, Pa. (1958).  
Simmons, L. D., "Critical Thickness of Insulation Accounting for Variable Convection Coefficient and Radiation Loss," *J. Heat Transfer*, **98**, 150 (1976).  
Sparrow, E. M., "Re-examination and Correction of the Critical Radius for Radial Heat Conduction," *AIChE J.*, **16**, 149 (1970).  
Yuge, T., "Experiments on Heat Transfer from Spheres Including Combined Natural and Forced Convection," *J. Heat Transfer*, **82**, 214 (1960).

Manuscript received July 22, 1977; revision received November 11, and accepted December 7, 1977.

## A Correlation for Solid Friction Factor in Vertical Pneumatic Conveying Lines

WEN-CHING YANG

Research and Development Center  
Westinghouse Electric Corporation  
Pittsburgh, Pennsylvania

Flowing gas-solids suspensions, commonly known as pneumatic conveying or pneumatic transport, have been widely practiced in the industry in loading and unloading of dry bulk materials and in distributing raw materials from the storage bins to the chemical reactors. Smooth operation of industrial chemical plants depends on reliable operation of these blood lines to supply the raw materials and to remove the wastes. Pneumatic transport is one of the known economic and feasible methods for performing these functions at low and medium pressure operations (up to  $\sim 25$  atm). Chemical reactions and physical operations such as drying can also be carried out in flowing gas-solids suspensions known as entrained bed reactors. Despite its importance, design of a pneumatic transport system remains an art rather than a science (Leung and Wiles, 1976). This paper will focus the discussion on only one aspect of the pneumatic transport, the frictional loss caused by solid particles in dilute phase vertical transport.

#### PRESSURE DROP IN A DILUTE PHASE VERTICAL PNEUMATIC CONVEYING LINE

There are numerous correlations available in the literature for predicting pressure drop in dilute phase vertical

pneumatic conveying as recently reviewed by Leung and Wiles (1976). The general recommended approach is to consider that the total pressure drop consists of three individual contributions due to acceleration, gravity, and wall friction:

$$\Delta P_T = \Delta P_A + \Delta P_S + \Delta P_F \quad (1)$$

The acceleration loss  $\Delta P_A$  can be evaluated as proposed by Yang and Keairns (1976)

$$\Delta P_A = \int_0^L \rho_p (1 - \epsilon) g dl + \int_0^L \frac{2f_g \rho_f U_f^2}{D} dl + \int_0^L \frac{f_p \rho_p (1 - \epsilon) U_p^2}{2D} dl + [\rho_p (1 - \epsilon) U_p^2]_{\text{at } L} \quad (2)$$

Beyond the acceleration region, the static head term can be expressed by

$$\Delta P_S = \rho_p (1 - \epsilon) g \cdot L \quad (3)$$

The friction term  $\Delta P_F$  is often separated into two terms due separately to fluid alone and to the effect of solid particles:

$$\Delta P_F = \Delta P_{Fg} + \Delta P_{Fs} \quad (4)$$

The friction due to conveying fluid alone is usually defined

following the Fanning equation as

$$\Delta P_{Fg} = \frac{2f_g \rho_f U_f^2 L}{D} \quad (5)$$

However, the friction due to solid particles  $\Delta P_{FS}$  is defined differently by many authors. It is important to distinguish these differences whenever the solid frictional loss is discussed.

#### DIFFERENT DEFINITIONS OF SOLID FRICTION FACTOR AND THEIR EVALUATION

Hariu and Molstad (1949), Konno and Saito (1969), Capes and Nakamura (1973), and Yousfi and Gau (1974) defined the solid friction factor following the Fanning equation based on solid particle velocity and the dispersed solid density:

$$\Delta P_{FS} = \frac{2f_s \rho_p (1 - \epsilon) U_p^2 L}{D} \quad (6)$$

Using this definition, Yousfi and Gau (1974) found  $f_s = 0.0015$  for polystyrene particles and  $f_s = 0.003$  for glass particles. The others found that  $f_s$  was inversely proportional to the particle velocity in the following form (Capes and Nakamura, 1973)

$$f_s = a U_p^{-b} \quad (7)$$

where  $a$  varies between 0.025 and 0.080 and  $b$  from 1 to 1.22. The data scattering are, however, usually larger than an order of magnitude.

Hinkle (1953), Wen and Galli (1971), and Yang (1973, 1976) defined the solid friction factor similarly with the following equation:

$$\Delta P_{FS} = \frac{f_p \rho_p (1 - \epsilon) U_p^2 L}{2D} \quad (8)$$

By definition,  $f_p$  is equal to  $4f_s$ . Hinkle (1953) suggested to evaluate the friction factor  $f_p$  from the following equation:

$$f_p = \frac{3\rho_f C_{Ds} D (U_f - U_p)^2}{2d_p \rho_p U_p^2} \quad (9)$$

Yang (1974) also proposed a correlation based on Ergun's equation to evaluate the  $f_p$  which will be discussed in detail later.

Jones et al. (1967) calculated the solid friction factor from an equation based on fluid density and fluid velocity:

$$\Delta P_{FS} = \frac{f'_s \rho_f U_f^2 L}{2D} \quad (10)$$

They also proposed the following empirical equation to evaluate the solid friction factor  $f'_s$

$$f'_s = 1.89 \times 10^{-6} \frac{A_o}{\phi^{1/2}} \theta^x \quad (11)$$

where

$$x = (6300/A_o)^{1/3} \quad \text{for } A_o > 6300 \text{ ft}^2/\text{ft}^3$$

and

$$x = 1 \quad \text{for } A_o < 6300 \text{ ft}^2/\text{ft}^3$$

Barth (1962) also defined the solid friction factor based on the fluid velocity as

$$\Delta P_{FS} = \frac{2f_s'' \rho_f U_f^2 L}{D} \cdot \frac{W_s}{W_f} \quad (12)$$

and correlated the solid friction factor with the fluid Froude number as

$$f_s'' = C Fr^{-1} \quad (13)$$

The constant  $C$  varies in different systems. Attempts have been made along this approach by Rose and Barnacle

(1957) to find a general correlation with empirical correction factors.

Other solid friction factor correlations by Richardson and McLeman (1960), Vogt and White (1948), Razumov (1962), and Mehta et al. (1957) have been reviewed by Boothroyd (1971).

It is interesting to note that no author has explicitly included the voidage ( $\epsilon$ ) as a correlation factor in all the solid friction factor correlations published so far, despite repeated appearances of various voidage functions in studies on packed beds (Ergun, 1952) and on fluidized beds (Wen and Yu, 1966; Richardson and Zaki, 1954). If voidage is an important parameter as suggested by Blake (1922) and Ergun (1952) in their pressure drop studies in packed beds, it should be equally important in dilute phase gas-solids transport where the number of solid particles and thus the number of collisions is directly proportional to  $(1 - \epsilon)$ .

#### DEVELOPMENT OF A NEW SOLID FRICTION FACTOR CORRELATION

Yang (1974) assumed that pneumatic conveying might behave similarly to moving beds if slip velocities were used in place of fluid velocities. From that preliminary assertion, data from Hariu and Molstad (1949) were plotted following Ergun's packed-bed equation. A unique correlation relating a modified solid friction factor and a modified Reynolds number was generated as shown in Equation (14):

$$f_p \frac{\epsilon^3}{(1 - \epsilon)} = 0.0206 \left[ (1 - \epsilon) \frac{(Re)_t}{(Re)_p} \right]^{-0.869} \quad (14)$$

This correlation was used in conjunction with the modified terminal velocity equation suggested earlier (Yang, 1973) to calculate solid particle velocity, particle acceleration length, acceleration pressure drop, and total pressure drop in vertical pneumatic conveying lines with reasonably accurate results (Yang, 1977).

Additional data by Konno and Saito (1969) and by Capes and Nakamura (1973) are correlated in the same

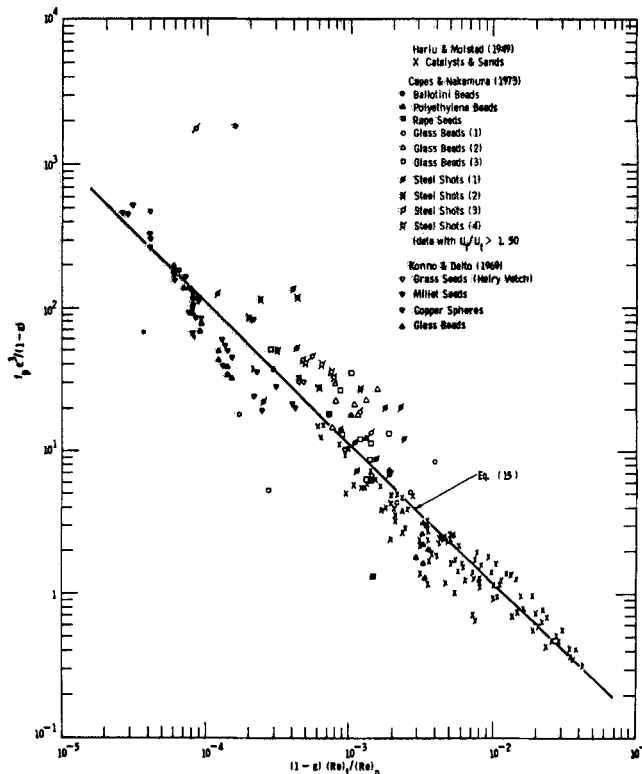


Fig. 1. Solid fraction factor correlation for viscous motion with slip.

TABLE 1. SUMMARY OF MATERIAL PROPERTIES AND CONVEYING CONDITIONS OF THE DATA USED IN THE CORRELATION FOR SOLID FRICTION FACTOR

Material	Average particle size, $\mu\text{m}$	Particle density, $\text{g}/\text{cm}^3$	Terminal velocity, $\text{m}/\text{s}$	Tube size, $\text{cm}$	References
Ottawa sand	503	2.64	3.90	0.678 and 1.35	Hariu and Molstad (1949)
Ottawa sand	357	2.64	2.80	0.678 and 1.35	Hariu and Molstad (1949)
Sea sand	274	2.64	2.20	0.678 and 1.35	Hariu and Molstad (1949)
Sea sand	213	2.71	1.59	0.678 and 1.35	Hariu and Molstad (1949)
Cracking catalyst	110	0.98	0.30	0.678 and 1.35	Hariu and Molstad (1949)
Glass beads	120	2.5	0.95	2.65 and 4.68	Konno and Saito (1969)
Glass beads	320	2.5	2.53	2.65 and 4.68	Konno and Saito (1969)
Glass beads	520	2.5	4.10	2.65 and 4.68	Konno and Saito (1969)
Glass beads	1 050	2.5	8.00	2.65 and 4.68	Konno and Saito (1969)
Copper spheres	120	8.9	2.20	2.65 and 4.68	Konno and Saito (1969)
Copper spheres	270	8.9	5.00	2.65 and 4.68	Konno and Saito (1969)
Copper spheres	530	8.9	9.80	2.65 and 4.68	Konno and Saito (1969)
Grass seeds	3 250	1.35	10.00	2.65	Konno and Saito (1969)
Millet seeds	1 440	1.44	7.13	2.65	Konno and Saito (1969)
Steel shot	260	7.51	3.99	7.62	Capes and Nakamura (1973)
Steel shot	530	7.85	8.26	7.62	Capes and Nakamura (1973)
Steel shot	1 200	7.70	14.91	7.62	Capes and Nakamura (1973)
Steel shot	2 340	7.70	22.93	7.62	Capes and Nakamura (1973)
Glass bead	1 080	2.90	7.96	7.62	Capes and Nakamura (1973)
Glass bead	1 780	2.90	11.31	7.62	Capes and Nakamura (1973)
Glass bead	2 900	2.86	15.43	7.62	Capes and Nakamura (1973)
Polyethylene	3 400	0.91	9.15	7.62	Capes and Nakamura (1973)
Rape seed	1 780	1.09	6.49	7.62	Capes and Nakamura (1973)

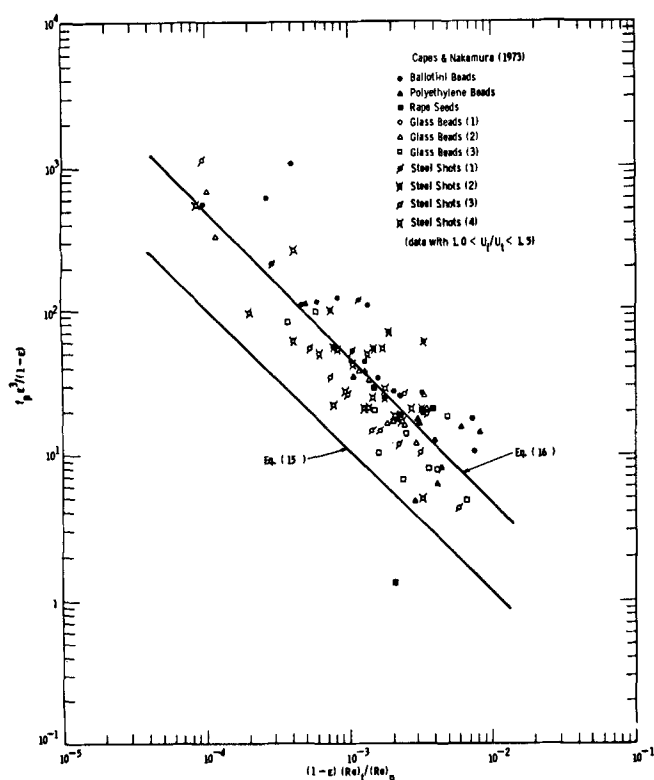


Fig. 2. Solid friction factor correlation for aggregative type of flow.

way as shown in Figure 1. The result of least-square curve fitting gives the following equation:

$$f_p \frac{\epsilon^3}{(1 - \epsilon)} = 0.0126 \left[ (1 - \epsilon) \frac{(Re)_t}{(Re)_p} \right]^{-0.979} \quad (15)$$

Slightly different numerical coefficients are obtained in

Equation (15) as compared to those in Equation (14). Equation (15) is suggested to replace Equation (14) for future application. The properties of the materials tested and the operating ranges used by different investigators are summarized in Table 1.

Only those data (by Capes and Nakamura, 1973) with  $U_f/U_t > 1.5$  are included in Figure 1. Those with  $U_f/U_t < 1.5$  are plotted in Figure 2. The numerical coefficients for the data in Figure 2 are evaluated by the least-squares method to give

$$f_p \frac{\epsilon^3}{(1 - \epsilon)} = 0.0410 \left[ (1 - \epsilon) \frac{(Re)_t}{(Re)_p} \right]^{-1.021} \quad (16)$$

Apparently, there are two separate flow regimes; however, the choice of  $U_f/U_t = 1.5$  here for distinction between the two different flow patterns is purely arbitrary.

Konno and Saito (1969) also observed two different types of particle flow during their experiments. They called them parallel flow and random flow. In parallel flow regime, the particles traveled relatively parallel to the pipe axis. Large numbers of collisions between the solid particles and the pipe wall were observed in the random flow regime. The pressure drops for these two types of flow showed large differences, and the pressure drop in the random flow regime was difficult to reproduce. Only the parallel flow data are included in their paper. Equations (2) and (3) and Figures 4 and 6 in their paper were used to back out the solid friction factor and the slip velocity used in the present correlation. For data which the slip velocities could not be found in Figure 6 of their paper, the slip velocities were assumed to equal the terminal velocities of the solid particles.

Because of the sensitivity of voidage and particle velocity on the correlation, only those literature data with direct experimental measurements of both the voidage and the particle velocity (or the slip velocity) in the transporting lines are included in the development of the correlation.

## DISCUSSION AND CONCLUSION

From the experimental evidences available so far, there are apparently at least two types of flow in dilute phase vertical pneumatic transport with large difference in pressure drop. One type of flow may be similar to the parallel flow described by Konno and Saito (1969). In this flow regime, the macroscopic scale of motion of particles is small (though the collisions among the individual particles may be significant), such that as far as the solid particles are concerned, it is analogous to viscous motion with slip (Soo, 1967). This type of flow can be correlated based on analysis of viscous motion of fluid through a packed bed as in Figure 1 and Equation (15) using slip velocity in place of fluid velocity.

At lower fluid velocities or in large diameter pipes, the solid particles may tend to segregate into clusters. This aggregate type of flow is especially true at fluid velocities close to the choking condition where large scale recirculation of particles occurs as observed by Capes and Nakamura (1973). A plausible model was proposed by Nakamura and Capes (1973). The transition criteria between these two flow regimes is still unknown. Separation of Capes and Nakamura (1973) data into two groups at  $U_f/U_t = 1.5$  as in Figures 1 and 2 is purely arbitrary. It is recommended that Equation (15) be used to calculate the solid friction factor in dilute phase vertical pneumatic transport, except in cases where transporting velocities are close to the terminal velocities of the solid particles. Equation (16) should be used in those conditions.

## ACKNOWLEDGMENT

The generosity of Drs. C. E. Capes and K. Nakamura who graciously supplied the original data of their work published in *Can. J. Chem. Eng.*, (51, 31 (1973)) is gratefully acknowledged.

## NOTATION

$a$	= empirical constant
$A_o$	= surface area of solid particles per unit volume
$b$	= empirical constant
$c$	= empirical constant
$C_{DS}$	= drag coefficient on a single particle
$D$	= inside diameter of conveying lines
$d_p$	= mean particle diameter
$f_g$	= gas friction factor as defined in Equation (5)
$f_p$	= solid friction factor, $f_p = 4f_s$
$f_s$	= solid friction factor as defined in Equation (6)
$f_s'$	= solid friction factor as defined in Equation (10)
$f_s''$	= solid friction factor as defined in Equation (12)
$g$	= gravitational acceleration
$l$	= acceleration length
$L$	= length of pipe beyond the acceleration section
$\Delta P_A$	= acceleration pressure drop
$\Delta P_F$	= total frictional pressure drop
$\Delta P_{Fg}$	= frictional pressure drop due to gas
$\Delta P_{Fs}$	= frictional pressure drop due to solid particles
$\Delta P_S$	= pressure drop due to static head
$\Delta P_T$	= total pressure drop in a line
$(Re)_p$	= Reynolds number defined as $d_p(U_f - U_p)\rho_f/\mu$
$(Re)_t$	= Reynolds number defined as $d_p U_t \rho_f/\mu$
$U_o$	= superficial fluid velocity based on empty column cross section
$U_f$	= actual fluid velocity defined as $U_o/\epsilon$
$U_p$	= actual particle velocity
$U_t$	= terminal velocity of a single particle
$W_f$	= gas flow rate
$W_s$	= solid flow rate

## Greek Letters

$\rho_f$	= fluid density
$\rho_p$	= particle density
$\mu$	= fluid viscosity
$\epsilon$	= voidage in transporting lines
$\phi$	= surface shape factor of solid particles
$\theta$	= specific solid loading, $W_s/W_f$

## LITERATURE CITED

- Barth, W., "Flow Problems with Mixtures of Gases and Entrained Solid Particles," *Eng. Digest*, **23**, 81 (1962).
- Blake, F. E., "The Resistance of Packing to Fluid Flow," *Trans. Am. Inst. Chem. Engrs.*, **14**, 415 (1922).
- Boothroyd, R. G., *Flowing Gas-Solids Suspensions*, PP141-150, Chapman and Hall, Ltd., London, England (1971).
- Capes, C. E., and K. Nakamura, "Vertical Pneumatic Conveying—An Experimental Study with Particles in the Intermediate and Turbulent Flow Regimes," *Can. J. Chem. Eng.*, **51**, 31 (1973).
- Ergun, S., "Fluid Flow Through Packed Columns," *Chem. Eng. Progr.*, **48**, No. 2, 89 (1952).
- Hariu, O. H., and M. C. Molstad, "Pressure Drop in Vertical Tubes in Transport of Solids by Gases," *Ind. Eng. Chem.*, **41**, 1148 (1949).
- Hinkle, B. L., "Acceleration of Particles and Pressure Drops Encountered in Horizontal Pneumatic Conveying," Ph.D. thesis, Ga. Inst. Technol., Atlanta (1953).
- Jones, J. H., W. G. Braun, T. E. Daubert, and H. D. Allendorf, "Estimation of Pressure Drop for Vertical Pneumatic Transport of Solids," *AIChE J.*, **13**, 608 (1967). Complete data have been deposited as document 9358 with the American Documentation Institute, Library of Congress, Washington 25, D.C.
- Konno, H., and S. Saito, "Pneumatic Conveying of Solids Through Straight Pipes," *J. Chem. Eng. Japan*, **2**, No. 2, 211 (1969).
- Leung, L. S., and R. J. Wiles, "A Quantitative Design Procedure for Vertical Pneumatic Conveying Systems," *Ind. Eng. Chem. Process Design Develop.*, **15**, No. 4, 553 (1976).
- Mehta, N. C., J. M. Smith, and E. W. Comings, "Pressure Drop in Air-Solid Flow Systems," *Ind. Eng. Chem.*, **49**, No. 6, 986 (1957).
- Nakamura, K., and C. E. Capes, "Vertical Pneumatic Conveying: A Theoretical Study of Uniform and Annular Particle Flow Models," *Can. J. Chem. Eng.*, **51**, 39 (1973).
- Razumov, I. M., "Calculating Pneumatic Transport of Solid Catalysts," *Intern. Chem. Eng.*, **2**, 539 (1962).
- Richardson, J. F., and M. McLeman, "Pneumatic Conveying," *Trans. Inst. Chem. Engrs.*, **38**, 25 (1960).
- Richardson, J. F., and W. N. Zaki, "Sedimentation and Fluidization: Part I," *ibid.*, **32**, 35 (1954).
- Rose, H. E., and H. E. Barnacle, "Flow of Suspensions of Non-Cohesive Spherical Particles in Pipes," *The Engineer* (June, 1957).
- Soo, S. L., *Fluid Dynamics of Multiphase Systems*, p. 216, Blaisdell Publication, Inc., Waltham, Mass. (1967).
- Vogt, E. G., and R. R. White, "Friction in the Flow of Suspension," *Ind. Eng. Chem.*, **40**, No. 9, 1731 (1948).
- Wen, C. Y., and A. F. Galli, "Dilute Phase Systems," in *Fluidization*, Davidson and Harrison, ed., Academic Press, New York (1971).
- Wen, C. Y., and Y. H. Yu, "Mechanics of Fluidization," *Chem. Eng. Progr. Symposium Ser. No. 62*, **62**, 101 (1966).
- Yang, W. C., "Estimating the Solid Particle Velocity in Vertical Pneumatic Conveying Lines," *Ind. Eng. Chem. Fundamentals*, **12**, 349 (1973).
- , "Correlations for Solid Friction Factors in Vertical and Horizontal Pneumatic Conveyings," *AIChE J.*, **20**, 605 (1974).
- , and D. L. Keairns, "Estimating the Acceleration Pressure Drop and the Particle Acceleration Length in Vertical and Horizontal Pneumatic Transport Lines," *Proc. Pneumotransport 3*, Organized by BHRA Fluid Eng., Bath, England (1976).
- Yang, W. C., "A Unified Theory on Dilute Phase Pneumatic Transport," *J. Powder Bulk Solids Tech.*, **1**, 89 (1977).

## Gas Absorption Accompanied by a Second-Order Chemical Reaction Modeled According to the Danckwerts Surface Renewal Theory

ERIK R. MATHERON

and

ORVILLE C. SANDALL

Department of Chemical and  
Nuclear Engineering  
University of California  
Santa Barbara 93106

In the work reported here, gas absorption accompanied by a second-order irreversible reaction was modeled by the Danckwerts surface renewal theory. The partial differential equations describing the unsteady state diffusion and reaction of gas A and liquid-phase reactant B were solved numerically, and the results of these numerical calculations were integrated over time according to the surface renewal theory to obtain the enhancement factor. The stoichiometry considered in this work,  $A + 2B \rightarrow \text{products}$ , corresponds to the industrially important case of carbon dioxide absorption in aqueous hydroxide solutions.

Solutions to this problem based on the penetration theory have been obtained by Brian et al. (1961) and by Pearson (1963). The predicted enhancement factors of this work were found to differ only slightly from those calculated from the Higbie penetration theory. The numerical results were also found to be in good agreement with the approximate analytical formula of DeCoursey (1974) over a wide range in the parameters. The DeCoursey equation is a simple, explicit relationship for the enhancement factor based on the Danckwerts model.

### DEVELOPMENT OF NUMERICAL SOLUTION

The physical situation for the absorption of gas A followed by subsequent irreversible reaction with liquid phase reactant B ( $A + 2B \rightarrow \text{products}$ ) can be represented by a pair of time-dependent diffusion equations:

$$\frac{\partial C_A}{\partial t} = D_A \frac{\partial^2 C_A}{\partial x^2} - k C_A C_B \quad (1)$$

$$\frac{\partial C_B}{\partial t} = D_B \frac{\partial^2 C_B}{\partial x^2} - 2k C_A C_B \quad (2)$$

The initial boundary conditions are taken to be:

$$t = 0 \quad x > 0 \quad C_B = C_{Bi} \quad C_A = 0$$

$$t > 0 \quad x = 0 \quad \frac{\partial C_B}{\partial x} = 0 \quad C_A = C_{Ao} \quad (3)$$

$$t > 0 \quad x \rightarrow \infty \quad C_B = C_{Bi} \quad C_A = 0$$

Equations (1) and (2) with boundary conditions given by Equations (3) were solved numerically in dimensionless form using the Crank-Nicolson finite-difference procedure.

The amount of gas A absorbed into the liquid per unit area in contact time  $\tau$  is given by

$$Q(\tau) = D_A \int_0^\tau \left( \frac{-\partial C_A}{\partial x} \right)_{x=0} dt \quad (4)$$

The enhancement factor based on a penetration theory model is obtained by dividing  $Q(\tau)$  by the amount of gas absorbed per unit area for physical absorption as predicted by penetration theory:

$$E_P(\tau) = Q(\tau) \sqrt{\frac{\pi}{D_A \tau}} / (2C_{Ao}) \quad (5)$$

Thus, the enhancement factor may be calculated from the results of the numerical calculations through Equations (4) and (5). Figure 1 shows some typical results of the calculations for  $D_B/D_A = 1$  and for  $0.2 \leq C_{Bi}/C_{Ao} \leq 10.0$ .

### ENHANCEMENT FACTOR BASED ON SURFACE RENEWAL THEORY

Pearson (1963) has obtained a large time asymptotic solution to Equations (1) and (2). Equation (6) expresses the rate of gas absorption at time  $\tau$  per unit area as determined from the large time asymptotic solution of Pearson: